Cognition 146 (2016) 229–239

Contents lists available at ScienceDirect

Cognition

journal homepage: www.elsevier.com/locate/COGNIT

Running the number line: Rapid shifts of attention in single-digit arithmetic

Romain Mathieu [a,b,*], Audrey Gourjon [a], Auriane Couderc [a], Catherine Thevenot [b,c], Jérôme Prado [a]

[a] Laboratoire Langage, Cerveau et Cognition (L2C2), Centre National de la Recherche Scientifique (CNRS) & Université Lyon 1, 69675 Bron cedex, France
[b] Faculté de Psychologie et des Sciences de l'Education (FPSE), Université de Genève, 1205 Genève 4, Switzerland
[c] Institut de Psychologie, Université de Lausanne, 1015 Lausanne, Switzerland

ARTICLE INFO

Article history:
Received 26 February 2015
Revised 30 July 2015
Accepted 4 October 2015

Keywords:
Arithmetic
Procedure
Retrieval
Attention
Mental number line
Arithmetic sign

ABSTRACT

It has been recently proposed that adults might solve single-digit addition and subtraction problems by rapidly moving through an ordered representation of numbers. In the present study, we tested whether these movements manifest themselves by on-line shifts of attention during arithmetic problem-solving. In two experiments, adult participants were presented with single-digit addition, subtraction and multiplication problems. Operands and operator were presented sequentially on the screen. Although both the first operand and the operator were presented at the center of the screen, the second operand was presented either to the left or to the right side of space. We found that addition problems were solved faster when the second operand appeared to the right than to the left side (Experiments 1 & 2). In contrast, subtraction problems were solved faster when the second operand appeared to the left than to the right side (Experiment 1). No operation-dependent spatial bias was observed in the same time window when the second operand was zero (Experiment 1), and no bias was observed when the operation was a multiplication (Experiment 2). Therefore, our results demonstrate that solving single-digit addition and subtraction, but not multiplication, is associated with horizontal shifts of attention. Our findings support the idea that mental movements to the left or right of a sequential representation of numbers are elicited during single-digit arithmetic.

1. Introduction

Mastering basic arithmetic is a major goal of elementary education and an essential first step toward higher-level mathematical abilities. Therefore, the strategies used by skilled adults to solve simple arithmetic problems have been the focus of a large body of literature over the past 40 years (Ashcraft and Guillaume, 2009, for a recent review). Using verbal reports and chronometric data, studies have converged to indicate that answers of simple arithmetic problems (such as single-digit addition, subtraction and multiplication) can either be retrieved from long-term memory (Campbell & Xue, 2001; Geary, Frensch, & Wiley, 1993; LeFevre, Sadesky, & Bisanz, 1996) or calculated using algorithmic procedures (e.g., counting, decomposition) (Barrouillet, Mignon, & Thevenot, 2008; Cooney, Swanson, & Ladd, 1988; Robinson, 2001; Seyler, Kirk, & Ashcraft, 2003). Typically, algorithmic procedures are seen as slow and effortful, whereas direct retrieval is considered fast and efficient. Therefore, there is a relative consensus in the literature that effective arithmetic learning is characterized by a shift from procedural to retrieval strategies (Ashcraft, 1982, 1992; Ashcraft & Guillaume, 2009; Geary, 1994; Siegler, 1996; Siegler & Shrager, 1984). In other words, the repetitive co-occurrence of a given problem with its answer during childhood would lead to a progressive association between that particular problem and answer in long-term memory (Geary & Burlingham-Dubree, 1989; Logan, 1988; Siegler & Shipley, 1995). The result is that skilled adults would not recruit procedural knowledge but largely rely on direct retrieval when solving simple arithmetic problems (Campbell & Xue, 2001; Geary et al., 1993). Algorithmic procedures would be mostly engaged when solving less practiced problems for which there is weak association between operands and answer (e.g., large problems) (LeFevre et al., 1996; Núñez-Peña, Gracia-Bafalluy, & Tubau, 2011; Thevenot, Barrouillet, & Fayol, 2001; Thevenot, Fanget, & Fayol, 2007).

Recently, a study cast doubt on this consensus. Using a priming paradigm, Fayol and Thevenot (2012) showed that skilled adults were faster at solving even very simple addition and subtraction

* Corresponding author at: Laboratoire Langage, Cerveau et Cognition, Centre National de la Recherche Scientifique (CNRS) & Université Lyon 1, 67 Boulevard Pinel, 69675 Bron cedex, France.

E-mail address: romain.mathieu@isc.cnrs.fr (R. Mathieu).

http://dx.doi.org/10.1016/j.cognition.2015.10.002
0010-0277/

problems (e.g., 3 + 2, 3 − 2) when the operation sign was presented 150 ms prior to the operands than when it was presented at the same time (see also Roussel, Fayol, & Barrouillet, 2002). Because no such priming was observed for single-digit multiplication, the effect appears to be operation-specific and may reflect the pre-activation of fast and automated procedures that could subsequently be used to solve addition and subtraction (but not multiplication) problems. Therefore, unlike what has been widely assumed in the past decades, procedural knowledge may still be recruited for solving even very simple addition and subtraction problems in skilled adults. Such procedural knowledge might not be recruited when solving multiplication problems, most likely because associations between operands and answers are explicitly learned by rote in school and only retrieved from long-term memory (Dehaene & Cohen, 1995).

A fundamental question arising from the findings of Fayol and Thevenot (2012) concerns the nature of the automated procedures that would be associated with single-digit addition and subtraction. It has been recently proposed that such procedures could take the form of a "process of rapid scrolling through an easily accessible and overlearned representation stored in long-term memory" (Barrouillet & Thevenot, 2013, p. 43). This proposal is consistent with the fact that solution times of even very small addition problems linearly increase as a function of operand size in adults (i.e., solution time increases with the distance between the original value and the value corresponding to the sum) (Barrouillet & Thevenot, 2013; Groen & Parkman, 1972). It suggests that the step-by-step counting procedures used by children when learning arithmetic might not totally disappear but instead be replaced by automatized counting procedures in adults (Barrouillet & Thevenot, 2013; Fayol & Thevenot, 2012). More generally, this proposal harks back to the idea that a key change in acquiring arithmetic efficiency may involve a shift from slow informal counting procedures to compiled procedural knowledge (Baroody, 1983, 1984, 1994). Because such internalized procedures do not necessarily reach consciousness, it could lead participants to mistakenly report using retrieval (Anderson, 1983; Ric & Muller, 2012).

If solving simple addition and subtraction problems does indeed involve rapid movement along an ordered representation of numbers, there are good reasons to assume that this process and representation are spatial in nature. Indeed, a growing number of studies document a link between numerical cognition and space (for a recent review, see Fischer & Shaki, 2014a). For example, studies have found that numbers are associated with a spatial bias in manual responses: When participants compare the magnitude of numbers (or classify them as even or odd), small numbers are processed faster with the left hand than with the right hand whereas large numbers are processed faster with the right hand than with the left hand (Dehaene, Bossini, & Giraux, 1993; Wood, Willmes, Nuerk, & Fischer, 2008). Numbers also automatically induce spatial shifts of attention. Specifically, small numbers facilitate the detection of a subsequent target in the left half of visual field (hereafter referred to as left hemifield) while large numbers facilitate the detection of a subsequent target in the right half of visual field (hereafter referred to as right hemifield) (Fischer, Castel, Dodd, & Pratt, 2003). Overall, these effects indicate that participants may represent numbers as spatially ordered items along a mental number line (MNL), with smaller magnitudes on the left side and larger magnitudes on the right side (Dehaene et al., 1993; Hubbard, Piazza, Pinel, & Dehaene, 2005).

More recently, studies have suggested that such spatial biases are not restricted to numbers but might also be present in symbolic arithmetic (Fischer & Shaki, 2014a, 2014b). Most evidence for a link between symbolic arithmetic and space comes from studies on complex arithmetic (i.e., problems involving multi-digit numbers that are typically not thought to be retrieved from memory). For example, Knops, Viarouge, and Dehaene (2009) showed that adults generally overestimate the result of complex symbolic addition while they underestimate the results of complex symbolic subtraction, an effect called *operational momentum* (OM) effect. As suggested by some (Hubbard et al., 2005; Knops, Dehaene, Berteletti, & Zorzi, 2014; Knops, Zitzmann, & McCrink, 2013; Knops et al., 2009), the OM effect might indicate that participants solve addition and subtraction problems by shifting their attention rightward or leftward along the MNL. The OM effect might stem from the fact that participants might move "too far" to the right (or to the left) along the MNL when solving an addition (or a subtraction) problem, leading to an overestimation (or an underestimation) of the actual result. Two studies provide further support for this attentional shift hypothesis. First, using functional magnetic resonance imaging (fMRI), Knops, Thirion, Hubbard, Michel, and Dehaene (2009) showed that multi-digit addition and subtraction problems are associated with different patterns of brain activation in the posterior superior parietal lobule (PSPL), a region involved in visuo-spatial processing. They further showed that the pattern of brain activation associated with multi-digit addition in that region is similar to the pattern of activation associated with rightward saccades (in line with the idea that participants shift their attention to the right of the MNL when solving multi-digit addition problems). Second, Klein, Huber, Nuerk, and Möller (2014) recorded eye movements of participants while they had to locate the results of (predominantly) multi-digit addition and subtraction results on a given number line. Consistent with the attentional shift hypothesis, the authors found that participants moved their eyes to the right of their first fixation on the line when they located the results of addition problems, while they moved their eyes to the left of their first fixation when they located the results of subtraction problems. Overall, these studies suggest that the procedures used by skilled adults to solve complex arithmetic problems might involve mentally moving along a spatial MNL.

The idea that addition and subtraction would involve asymmetric shifts of attention along the MNL is broadly consistent with Barrouillet and Thevenot's proposal of moving along a representation of numbers (Barrouillet & Thevenot, 2013). Yet, it remains unknown whether these attentional shifts are elicited on-line during the resolution of simple arithmetic problems (i.e., problems involving single-digit numbers that are typically thought to be retrieved) and could provide the basis for the fast and automatic procedures hypothesized by Barrouillet and Thevenot (2013) and Fayol and Thevenot (2012). To our knowledge, only a few studies have investigated the link between space and simple arithmetic problem-solving.

First, Pinhas and Fischer (2008) asked participants to point to the results of single-digit arithmetic problems on a number line that was visually presented. For a same result (e.g., "6"), participants' pointing was biased to the right for an addition (e.g., 4 + 2) and to the left for a subtraction (e.g., 8 − 2). Therefore, this study indicates the presence of an OM in simple arithmetic. However, problems containing zero were associated with an even larger OM than other problems. This is inconsistent with the idea that the OM in that study stems from shifts of attention elicited by arithmetic calculation because addition and subtraction problems containing zero should not require any differential movement along the MNL. Thus, the authors proposed a "spatial competition account" according to which each component of an arithmetic problem (i.e., the operands, the operator and the result) leads to competing spatial activations along the MNL (Pinhas & Fischer, 2008). Another account posits that the OM might be accounted for by the heuristic "accepting more than the first operand" for addition and "accepting less than the first operand" for subtraction (Knops et al., 2009; McCrink & Wynn, 2009). Other alternative accounts that do not involve shifts along the MNL have been proposed as well (Chen & Verguts, 2012; Knops et al., 2009; McCrink, Dehaene, & Dehaene-Lambertz, 2007; Pinhas & Fischer, 2008).

Second, Masson and Pesenti (2014) asked participants to detect a target that was flashed either to the left or to the right of a central fixation point immediately after participants had provided the answer of a subtraction or addition problem. The results provide mixed evidence regarding a link between single-digit arithmetic and space. On the one hand, targets in the left hemifield were better detected than targets in the right hemifield after solving single-digit subtraction problems. This is consistent with the idea that subtraction problems are associated with leftward shifts of attention. On the other hand, no difference was found between hemifields after single-digit addition problems. This suggests that single-digit addition might not be associated with any shifts of attention. However, these results need to be interpreted with caution. First, the target always appeared *after* calculation was over (i.e., 450 ms after participants had provided the answer to the problem). Thus, the design might not be well suited for capturing a rapid shift of attention that would occur *during* the calculation process (i.e., before participants provide the answer). Second, the result size of arithmetic problems was small overall, and even smaller for subtraction (3.9 on average) than for addition (6.1 on average). Because small numbers are associated with the left side of space and the target was presented after participants provided the answer (Fischer et al., 2003), the leftward bias observed for subtraction might be due to the small magnitude of the result rather than to the calculation process.

Third, two recent studies attempted to demonstrate a link between single-digit arithmetic and space by examining interactions with hand or eye movements. Marghetis, Núñez, and Bergen (2014) measured hand trajectories of participants selecting the correct result of single-digit addition or subtraction problems on a computer screen with a mouse cursor. Correct results were displayed either to the left or to the right of the center of the screen. Compared to trajectories of hand movements performed when results were congruent to the predicted direction (i.e., left for subtraction and right for addition), overall trajectories of hand movements that were performed when results were incongruent (i.e., right for subtraction and left for addition) were deflected in the opposite direction. Because the authors directly compared congruent versus incongruent problems without distinguishing between addition and subtraction, it is unknown whether this bias was driven by one or the other operation (or both). Nonetheless, this study provides some important support for an association between single-digit arithmetic and space. Recently, Hartmann, Mast, and Fischer (2015) attempted to more directly measure online shifts of attention *during* simple arithmetic problem-solving. This was done by tracking eye movements of participants while they solved single-digit addition and subtraction problems. Unlike Marghetis et al. (2014), the results did not show any significant difference between addition and subtraction in the horizontal dimension (relative differences between operations were only observed in the vertical dimension and appeared to be mostly driven by a downward bias for subtraction, see their Fig. 2B). Therefore, to our knowledge, there is no clear-cut evidence that horizontal shifts of attention occur *during* simple arithmetic problem-solving.

The goal of the present study was to test the hypothesis that simple arithmetic problem-solving is associated with horizontal shifts of attention that may reflect rapid shifts along the MNL (Barrouillet & Thevenot, 2013). We asked adult participants to solve single-digit arithmetic problems in which operands and arithmetic sign were displayed sequentially on a screen (see Fig. 1). In each trial, the first operand (O1) and the arithmetic sign were presented one after the other at the center of the screen. The arithmetic sign then disappeared and was followed by the second operand (O2), which was presented either to the left or to the right of the center of the screen. Participants were asked to verbally solve the problem as fast and accurately as they could. The position of O2 on the screen was irrelevant for the task. Nonetheless, if simple arithmetic problem-solving is associated with horizontal shifts of attention, performance should be facilitated when O2 appears in the hemifield congruent to the attentional shift. More specifically, if addition problems are accompanied by rightward shifts of attention, response time (RT) should be faster when O2 appears in the right compared to the left hemifield. Inversely, if subtraction problems are accompanied by leftward shifts of attention, RT should be faster when O2 appears in the left compared to the right hemifield.

2. Experiment 1

2.1. Method

2.1.1. Participants

Forty students from the University of Lyon volunteered to participate in this experiment. All were native French speakers and had normal or corrected-to-normal vision. Four participants were excluded from further analysis because of technical recording issues ($n = 2$) or non-compliance with the instructions during the whole experiment ($n = 2$). Data from two participants were further removed because they were outliers (as determined by overall dRTs that differed by more than 2 Standard Deviation [SD] from the mean of all participants, see below). The remaining 34 participants (20 females; 30 right-handed) were aged between 19 and 31 years (M = 21.8, SD = 2.7 years). The experiment was performed in accordance with the ethical standards established by the Declaration of Helsinki.

2.1.2. Stimuli

The main arithmetic problems were constructed following the criteria used by Fayol and Thevenot (2012). All pairs of non-identical operands between 1 and 5 [(2,1); (3,1); (3,2); (4,1); (4,2); (4,3); (5,1); (5,2); (5,3); (5,4)] were used to construct small addition and subtraction problems. All pairs of non-identical operands between 5 and 9 [(6,5); (7,5); (7,6); (8,5); (8,6); (8,7); (9,5); (9,6); (9,7); (9,8)] were used to construct large addition and subtraction problems. The number 5 was used in both categories to ensure that there were as many small problems as there were large problems. The largest operand was O1 in all addition and subtraction problems. This ensured that (i) the results of subtraction problems were always positive and that (ii) the type of operation could not be predicted from the first operand (because of the sequential presentation of the problems, see below). There were 20 addition problems (10 small/10 large) and 20 subtraction problems (10 small/10 large). Following a reviewer's suggestion, some participants ($n = 15$ in the final sample) were also presented with 18 problems involving zero as O2 (9 addition and 9 subtraction problems). These *zero-problems* were constructed with the following pairs of operands: (1,0); (2,0); (3,0); (4,0); (5,0); (6,0); (7,0); (8,0); (9,0). Solving a problem with zero as O2 should not require any shift along the MNL. Thus, any operation-dependent spatial bias observed for the main problems that would be due to differential movements along the MNL should not be found with zero-problems. See Appendix A for a full list of arithmetic problems.

2.1.3. Task and procedure

Participants were asked to solve arithmetic problems displayed on a computer screen. For each problem, operands and arithmetic sign were presented sequentially (Fig. 1). A trial started with the presentation of a white central fixation dot for 500 ms, immediately followed by O1 for an additional 500 ms. After a first delay of 500 ms, the arithmetic sign (+ or −) was flashed for 150 ms at the center of

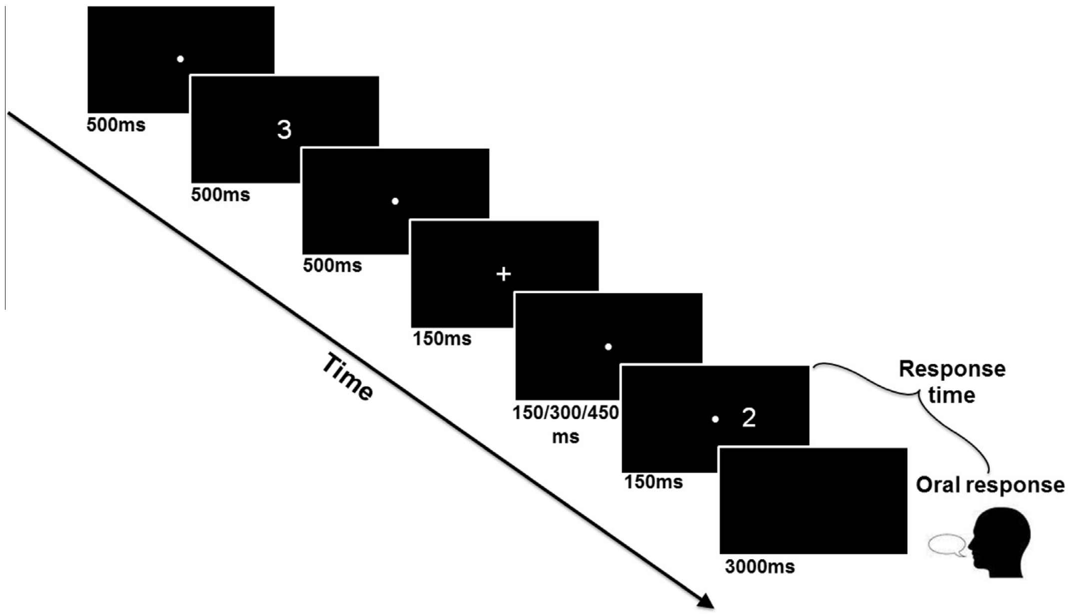

Fig. 1. Sequence and timing of a sample trial. Arithmetic problems were presented sequentially on a screen. Both the first operand and the operator were presented at the center of the screen. The second operand was flashed either in the left or the right hemifield. Participants had a maximum of 3000 ms to give aloud their response through a headset microphone. Response time corresponded to the delay between the onset of the second operand and the onset of the oral answer.

the screen. This was followed by a second delay separating the arithmetic sign from O2, which was flashed for 150 ms either 5° to the left or 5° to the right of the center of the screen. Previous studies have found priming effects associated with arithmetic signs using stimulus onset asynchronies (SOAs) of 150 ms (Fayol & Thevenot, 2012) and 300 ms (Roussel et al., 2002). Interactions between arithmetic processing and space have further been observed with SOAs of 450 ms (Masson & Pesenti, 2014). Therefore, we varied the delay between the arithmetic sign and O2 in the present experiment, such that the SOA was of 150, 300, or 450 ms. Participants were given 3000 ms to say their answer aloud as quickly and accurately as possible before the beginning of the next trial.

Each problem was presented once for each SOA (150, 300, 450 ms) and once for each side of appearance of O2 (left, right). Therefore, each problem was presented 6 times across the whole experiment. Participants performed the task in 4 successive blocks, with an equal number of trials in each. In each block, trials were pseudorandomly ordered so that no more than three problems of the same type could appear consecutively. The order of blocks was counter-balanced between subjects. The experiment started with 8 practice trials. These practice trials included tie problems (e.g., 2 + 2) and problems involving both small and large digits (e.g., 3 + 9). For those participants who were not presented with zero-problems in the main experiment, zero-problems (e.g., 5 × 0) were also included in the practice session. The whole experiment lasted between 25 and 35 min.

During the entire experiment, participants were seated at 44 cm from a 15″ computer screen with their head stabilized by a chin rest and frontal support to minimize head movements. Problems were displayed in white Times New Roman 36-point font on a black background. Participants were instructed to keep their eyes fixated on the center of the screen and not to make any eye movements. A webcam was positioned at the top of the computer screen. Although this was not true, participants were told that eye position will be monitored during the entire experiment to ensure that they keep fixation. The experiment was controlled by the DmDX software (Forster & Forster, 2003). Response times (RTs) were recorded through a headset microphone and corresponded to the period between the presentation of O2 and the onset of the answer. For each trial and participant, RT was checked off-line and manually adjusted if necessary with CheckVocal (Protopapas, 2007).

2.2. Results

RT data were normalized using a logarithmic transformation prior all analyses to improve the conformity of the data to the standard assumptions of parametric testing (Howell, 2011). The analysis was performed on correct trials only (i.e., 94.9% of the trials). Trials in which the answer was not recorded and outlier trials (trials with a RT smaller than 200 ms or a RT greater than 2 SDs from the mean for each participant) were removed from the analyses (this corresponded to a further 4.7% of the trials). For each participant, operation and problem size, we subtracted the mean RT of trials in which O2 appeared on the right from the mean RT of trials in which O2 appeared on the left. This difference in RT (dRT) served as dependent variable in the following analyses. However, for the sake of completeness, raw mean RTs as a function of Problem size and SOA are also presented in Tables 1 and 2. In what follows, we

Table 1
Mean RT (and SD) as a function of Operation, Problem size and SOA for main arithmetic problems in Experiment 1.

	Addition			Subtraction		
	150 ms	300 ms	450 ms	150 ms	300 ms	450 ms
Small	796 (129)	783 (131)	782 (119)	875 (156)	841 (146)	848 (141)
Large	1326 (349)	1270 (345)	1267 (340)	946 (186)	922 (196)	906 (171)

Table 2
Mean RT (and SD) as a function of Operation and SOA for zero-problems in Experiment 1.

	Addition			Subtraction		
	150 ms	300 ms	450 ms	150 ms	300 ms	450 ms
Zero	729 (111)	714 (105)	727 (113)	748 (120)	730 (116)	745 (111)

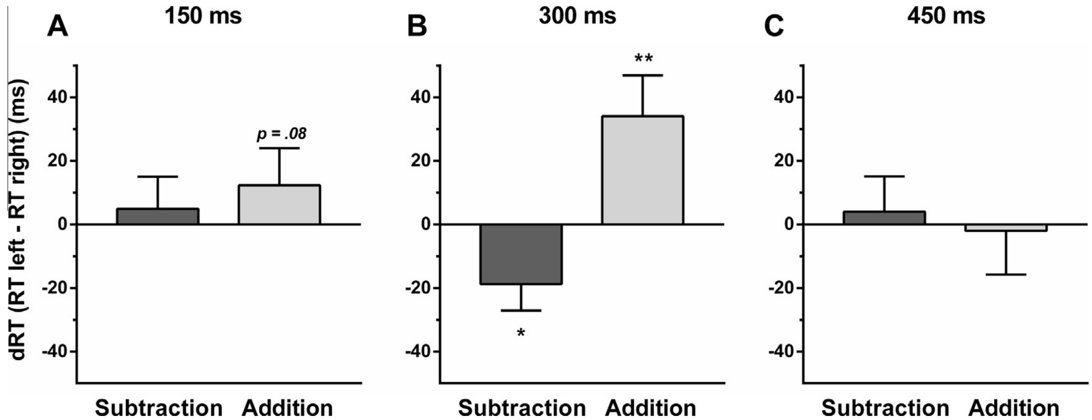

Fig. 2. dRT as a function of Operation (Subtraction, Addition) for main arithmetic problems with the 150 ms SOA (A), 300 ms SOA (B) and 450 ms SOA (C) in Experiment 1. Error bars represent standard error of the mean (SEM). *$p < .05$; ** $p < .01$.

present findings regarding the main arithmetic problems (i.e., problems involving pairs of non-zero operands) in all participants, before examining zero-problems in a subset of the participants. Data were analyzed using repeated-measures ANOVAs. For follow-up *t*-tests, one-tailed *p* values are reported because our hypotheses were directional (i.e., we anticipated a rightward bias for addition and a leftward bias for subtraction). *P* values less than 0.05 were considered to be significant.

2.2.1. Main problems

dRTs associated with all main problems were entered into a $2 \times 2 \times 3$ analysis of variance (ANOVA) with the within-subject factors Operation (Addition, Subtraction), Problem size (Small, Large), and SOA (150 ms, 300 ms, 450 ms). Although the interaction between Operation and SOA was not reliable ($F(2,66) = 2.23$, $MSe = .009$, $\eta^2 = .014$, $p = .115$), the main effect of Operation was significant ($F(1,33) = 4.33$, $MSe = .006$, $\eta^2 = .009$, $p < .05$). Across all SOAs, dRT was more positive for addition (15 ms) than for subtraction (−3 ms). Follow-up ANOVAs conducted separately for each SOA (with the within-subject factors Operation and Problem size) revealed that this effect was driven by the 150 ms and 300 ms SOAs. With the 150 ms SOA, although the main effect of Operation was not significant ($F(1,33) = .33$, $MSe = .008$, $\eta^2 = .003$, $p = .57$), dRT tended to be positive for addition problems (dRT = 12 ms; $t_{33} = 1.40$, Cohen's $d_z = .24$, $p = .08$) whereas it was not different from 0 for subtraction (dRT = 5 ms; $t_{33} = .39$, Cohen's $d_z = .07$, $p = .35$) (Fig. 2A). Therefore, 150 ms after the arithmetic sign, addition problems tended to be solved faster when O2 was presented to the right than to the left, whereas no bias was observed for subtraction problems. With the 300 ms SOA, the main effect of Operation was significant ($F(1,33) = 11.16$, $MSe = .006$, $\eta^2 = .094$, $p < .01$). Specifically, dRT was reliably positive for addition (dRT = 34 ms; $t_{33} = 2.67$, Cohen's $d_z = .46$, $p < .01$) and reliably negative for subtraction (dRT = −19 ms; $t_{33} = 2.10$, Cohen's $d_z = .36$, $p < .05$) (Fig. 2B). Therefore, 300 ms after the arithmetic sign, addition problems were solved faster when O2 was presented to the right than to the left, whereas subtraction problems were solved faster when O2 was presented to the left than to the right. Finally, with the 450 ms SOA, the main effect of Operation did not reach significance ($F(1,33) = .03$, $MSe = .012$, $\eta^2 = .0005$, $p = .86$): dRT differed from 0 neither for addition nor for subtraction (addition: dRT = −2 ms; $t_{33} = .01$, Cohen's $d_z = .002$, $p = .50$; subtraction: dRT = 4 ms; $t_{33} = .30$, Cohen's $d_z = .05$, $p = .38$) (Fig. 2C). Overall, then, a rightward bias was observed for addition problems as early as 150 ms after the arithmetic sign, whereas a leftward bias was observed for subtraction problems as early as 300 ms after the sign. These biases appear to have faded away 450 ms after the sign.

Finally, the $2 \times 2 \times 3$ ANOVA did not reveal any main effect of Problem size ($F(1,33) = .30$, $MSe = .005$, $\eta^2 = .0005$, $p = .59$), but the interaction between Operation and Problem size was significant ($F(1,33) = 4.74$, $MSe = .010$, $\eta^2 = .015$, $p < .05$). This indicated that dRT was more negative for large (−14 ms) than small (8 ms) subtraction problems ($t_{33} = 2.29$, Cohen's $d_z = .39$, $p < .05$), whereas there was no difference between large (27 ms) and small (3 ms) addition problems ($t_{33} = 1.31$, Cohen's $d_z = .22$, $p = .10$). However, this interaction was only observed across all SOAs: Follow-up ANOVAs conducted separately for each SOA did not reveal any significant (or near-significant) interaction between Problem size and Operation (or main effect of Problem size) in any of the SOA.

2.2.2. Zero-problems

If the dRT difference observed between main addition and main subtraction problems 300 ms after the sign is due to differential movements along the MNL, it should not be observed when O2 is equal to zero (because no movement along the MNL should be elicited in this case). To test this hypothesis, we presented a subset of participants ($n = 15$) with problems in which O2 was zero (i.e., zero-problems). Unlike for the main arithmetic problems, we did not find any dRT difference between zero addition (24 ms) and zero subtraction (25 ms) problems with the 300 ms SOA ($t_{14} = .22$, Cohen's $d_z = .06$, $p = .41$). This lack of effect was not due to a lack of power because dRT was more positive for main addition (12 ms) than main subtraction (−20 ms) problems in the exact same participants ($t_{14} = 1.46$, Cohen's $d_z = .38$, $p = .08$). Thus, the operation-dependent spatial bias observed 300 ms after the arithmetic sign for the main problems appears to be restricted to problems in which O2 is greater than 0.

Interestingly, although there was no dRT difference between zero addition (−3 ms) and zero subtraction (−9 ms) problems with the 150 ms SOA ($t_{14} = .33$, Cohen's $d_z = .09$, $p = .37$), we found a significant difference for the 450 ms SOA ($t_{14} = 1.96$, Cohen's $d_z = .51$, $p < .05$). With that SOA, dRT was reliably positive for addition problems (dRT = 22 ms; $t_{14} = 2.67$, Cohen's $d_z = .69$, $p < .01$), but did not differ from 0 for subtraction problems (dRT = −2 ms; $t_{14} = .05$, Cohen's $d_z = .01$, $p = .48$). This effect was specific to zero problems because it was not observed for the main problems in the same participants (as in the whole group). Overall, these results indicate that zero-problems may be associated with an operation-dependent spatial bias to some extent. However, unlike for the main arithmetic problems, this effect is late developing and was only found 450 ms after the arithmetic sign.

2.3. Discussion

It has been recently proposed that adults might solve single-digit addition and subtraction problems by rapidly moving to the right or left of a MNL (Barrouillet & Thevenot, 2013). The purpose of this first experiment was to test whether solving single-digit addition and subtraction problems is associated with covert shifts of attention toward the right or left side of space. We designed a task in which operands and arithmetic sign were presented sequentially on a screen. Both O1 and the arithmetic sign were presented at the center of the screen. O2 was presented either in the left or in the right hemifield. Even though the location of O2 on the screen was irrelevant for the task, participants were faster at solving single-digit addition when O2 was presented in the right hemifield compared to the left. Conversely, participants were faster at solving single-digit subtraction when O2 was presented in the left hemifield compared to the right. These effects were relatively early developing (i.e., they were observed as early as 150 ms after the onset of the operator for addition and as early as 300 ms after the onset of the operator for subtraction) and had faded away 450 ms after the arithmetic sign. Up to 300 ms after the sign, the operation-dependent spatial bias was also restricted to operations that did not involve zero as O2.

Overall, these results demonstrate that solving single-digit addition and subtraction problems is associated with on-line horizontal shifts of attention. This is consistent with the idea that these problems activate procedures that may involve shifts to the right or left of a MNL (Barrouillet & Thevenot, 2013). Two previous studies suggest that such procedural knowledge is likely cued by the arithmetic sign (Fayol & Thevenot, 2012; Roussel et al., 2002). Fayol and Thevenot (2012) and Roussel et al. (2002) showed that the simple perception of the arithmetic sign 150 ms or 300 ms before a problem (addition or subtraction) could facilitate subsequent problem-solving, presumably by priming automatic procedures that would be used to solve these problems. It is possible that the arithmetic sign might cue the activation of a MNL, along which participants can shift their attention to move to the right or left of O1 (for addition and subtraction, respectively). Such shifts, however, should naturally only occur when O2 is greater than zero. In line with this claim, we did not find any differential association between operation and side of O2 for zero-problems within the time window in which we found effects for the main arithmetic problems (150 ms and 300 ms after the sign). Thus, the early spatial biases captured here are likely to reflect procedures relying on shifts along the MNL that would be engaged when computing the result of arithmetic problems.

There are two other aspects of our results that are worth discussing here. First, we found that the leftward bias associated with subtraction did not appear to emerge as early as the rightward bias associated with addition (300 ms after the sign for subtraction and 150 ms after the sign for addition). We speculate that the earlier onset of rightward versus leftward movements along the MNL may be due to the fact that addition may be more practiced than subtraction over the course of arithmetic education (e.g., addition is typically learned and practiced before subtraction in school), or even in everyday life. Thus, the rightward movements associated with addition problem-solving might end up being elicited slightly earlier than the leftward movements associated with subtraction problem-solving in adults. This superiority for rightward over leftward shifts might be further enhanced by reading habits (i.e., all of our participants read from left to right). Second, although there was no differential association between operation and side of O2 for zero-problems within the time window in which we found the effects for the main problems (150 ms and 300 ms after the sign), there was a greater rightward bias for zero addition than zero subtraction problems 450 ms after the arithmetic sign (where no effect was found for the main arithmetic problems). The lack of temporal overlap between this effect and the operation-dependent spatial bias observed for the main problems most likely indicate that they have a different origin. One possibility for explaining this effect is suggested by the spatial competition account proposed by Pinhas and Fischer (2008) (see also Pinhas, Shaki, & Fischer, 2014, 2015). There is accumulating evidence that both numbers (Fischer et al., 2003) and operators (Marghetis et al., 2014; Pinhas et al., 2014) have spatial associations, which seem to appear around 400 ms (at least for numbers, see Fischer et al., 2003). Thus, the effect observed here for zero problems 450 ms after the sign may be due to a late association between addition signs and the right side of space. This effect might be diluted in main arithmetic problems because non-zero numbers might induce a spatial activation that might compete with the spatial activation associated with the sign (Pinhas & Fischer, 2008). But such a competition should be absent for zero-problems because zero is not thought to be represented in the MNL (Brysbaert, 1995; Tzelgov, Ganor-Stern, & Maymon-Schreiber, 2009). Finally, the lack of leftward association for non-zero subtraction problems might be explained by the relatively weak association between subtraction signs and left side of space (see Pinhas et al., 2014, Experiment 1).

Overall, shifts along the MNL appear to be a good explanation for the effects observed 150 ms and 300 ms after the sign. We see, however, at least two other possible explanations for these effects. First, it is possible that with time and practice, participants associate arithmetic signs with simple heuristics. For example, Hartmann et al. (2015) proposed that "the principal role of space during mental arithmetic might be the activation of metaphorical magnitude concepts" (Hartmann et al. (2015), p. 6), such as "more is right" and "less is left". In other words, participants might shift their attention to the left or right side of space not because they move along the MNL, but simply because the appearance of the addition or subtraction sign makes them anticipate that the result of the problem will be smaller or larger than the first operand. This could "provide an intuitive check on rote or algorithmic calculation" (Marghetis et al., 2014). Second, it is also possible that these horizontal biases are due to differences in the size of the results between addition and subtraction. Indeed, results of addition problems were larger overall than results of subtraction problems and larger numbers are more strongly associated with the right side of space than smaller numbers. Thus, it remains conceivable that number-space associations might have driven the effect (Fischer et al., 2003).

Interestingly, examining whether multiplication problems are associated with attentional biases could be very informative for distinguishing between our main interpretation and these two other accounts. If attentional biases are due to movements along the MNL, no bias should be observed for multiplication problems. Fayol and Thevenot (2012) did not find any facilitation of problem-solving when a multiplication sign was presented before the occurrence of a multiplication problem. This indicates that multiplication signs do not activate any procedural knowledge and that the results of such problems are likely retrieved from long-term memory, as suggested by a large body of literature (Ashcraft, 1992; Campbell & Xue, 2001; Galfano, Rusconi, & Umiltà, 2003; Ischebeck et al., 2006; Rusconi, Galfano, Speriani, & Umiltà, 2004; Thibodeau, Lefevre, & Bisanz, 1996). In contrast, if participants have developed an association between the addition sign and a heuristic such as "more than the first operand", one should observe the same rightward bias for multiplication as for addition. This is because results of multiplication problems are always larger than the first operand (at least when the second operand is greater than 1). A similar rightward bias for multiplication should also be observed if attentional biases are driven by the size of the results. Indeed, when the same operands are used, results of multiplication problems are significantly larger than results of addition problems and should be even more strongly associated with the right side of space. In Experiment 2, we aimed to test between these hypotheses by presenting participants with single-digit multiplication problems using the same paradigm as in Experiment 1. Because the extension to multiplication problems leads to a large number of trials by participant, only addition and multiplication problems were presented to participants in this second experiment (both types of problems consisted in the exact same operands).

3. Experiment 2

3.1. Method

3.1.1. Participants

Twenty-four students from the University of Lyon volunteered to participate in this experiment. All were native French speakers and had normal or corrected-to-normal vision. Two participants were excluded from further analysis (one because of technical recording issues and one because of an error rate greater than 40%). The remaining 22 participants (10 females; all right-handed) were aged between 18 and 27 years (M = 21.8, SD = 2.3 years). The experiment was performed in accordance with the ethical standards established by the Declaration of Helsinki.

3.1.2. Stimuli

The same pairs of operands as those used in the main arithmetic problems of Experiment 1 (i.e., problems involving pairs of non-zero operands) were used to construct addition and multiplication problems. Unlike in Experiment 1, however, the absence of subtraction problems and of problems involving zero allowed us to present the largest operand as either first or second operand (see below). See Appendix B for a full list of arithmetic problems.

3.1.3. Task and procedure

The task, apparatus, and stimulus timing were identical to those in Experiment 1. However, because the largest rightward bias for addition problems was found with the 300 ms SOA in Experiment 1, we only presented problems with that particular SOA in Experiment 2. Each problem was presented once for each order of presentation of operand (first versus second position) and once for each side of appearance of O2 (left, right). Therefore, each problem was presented 4 times across the whole experiment for a total of 180 trials distributed in 3 successive blocks of 60 pseudorandomly ordered trials. The order of blocks was counter-balanced between subjects. The experiment started with 8 practice trials (constructed in the same manner as in Experiment 1) and lasted less than 20 min.

3.2. Results

As for Experiment 1, RT data were normalized using a logarithmic transformation prior all analyses (Howell, 2011). The analysis was performed on correct trials only (i.e., 89.4% of the trials). Trials in which the answer was not recorded and outlier trials (defined in the same was as in Experiment 1) were also removed from the analyses (this corresponded to a further 4.9% of the trials). dRT (difference in RT between problems in which O2 was on the left and problems in which O2 was on the right) served as dependent variable. However, for the sake of completeness, raw mean RTs as a function of Problem size and Operation are also reported in Table 3. Data were analyzed using repeated-measures ANOVAs. For follow-up *t*-tests, one-tailed *p* values are reported because our hypotheses were directional (i.e., we anticipated a rightward bias for addition and either a rightward or no bias for multiplication). *P* values less than 0.05 were considered to be significant.

Table 3
Mean RT (and SD) as a function of Operation and Problem size in Experiment 2.

	Addition	Multiplication
Small	826 (108)	811 (97)
Large	1359 (331)	1268 (215)

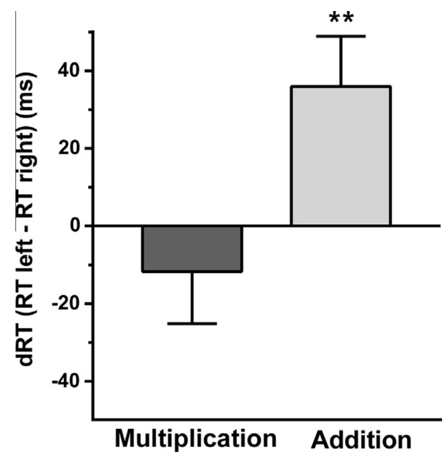

Fig. 3. dRT as a function of Operation (Multiplication, Addition) in Experiment 2. Error bars represent standard error of the mean (SEM). ** $p < .01$.

dRTs were entered into a 2 × 2 × 2 analysis of variance (ANOVA) with the within-subject factors Operation (Addition, Multiplication), Problem size (Small, Large), and Position of smallest operand (First, Second). The ANOVA only revealed a main effect of Operation ($F(1,21) = 6.69$, $MSe = .010$, $\eta^2 = .046$, $p < .05$) (Fig. 3). Follow-up *t*-tests revealed that dRT was positive for addition (dRT = 36 ms; $t_{21} = 3.01$, Cohen's $d_z = .64$, $p < .01$), whereas it did not differ from 0 for multiplication (dRT = −12 ms; $t_{21} = 0.79$, Cohen's $d_z = .17$, $p = .22$). No other main effect or interaction reached significance. Overall, these results clearly indicate that addition problems were solved faster when O2 was presented to the right than to the left, whereas the side of presentation of O2 did not affect the solving time of multiplication problems.

3.3. Discussion

The second experiment successfully replicated the results of the first experiment regarding addition problems. Specifically, participants were faster at solving single-digit addition when O2 appeared in the right hemifield compared to the left hemifield. However, the position of O2 did not appear to affect the solving time of multiplication problems. This is inconsistent with the proposal that attentional biases in addition problems are due to associations between arithmetic signs and a heuristic such as "more than the first operand", because results of multiplication problems are also larger than (or at least as large as) the first operand and the same heuristic could be applied. This is also inconsistent with the hypothesis that attentional biases are driven by the magnitude of the result because results of multiplication problems are larger than those of addition problems and should be even more strongly associated with the right side of space. Therefore, the attentional biases observed for the main (i.e., non-zero) subtraction (in Experiment 1) and addition (in Experiments 1 and 2) problems are more likely due to movements along the MNL that do not occur with multiplication problems. The results of the two experiments are discussed in details below.

4. General discussion

Several recent studies suggest that fast and automatized counting procedures are elicited when solving very simple addition and subtraction problems (Barrouillet & Thevenot, 2013; Fayol & Thevenot, 2012; Ric & Muller, 2012). These procedures have been posited to take the form of a "rapid scrolling through an easily accessible and overlearned representation stored in long-term memory" (Barrouillet & Thevenot, 2013; Fayol & Thevenot, 2012). The goal of the present experiments was to test whether such procedures might manifest themselves by on-line horizontal shifts of attention during problem-solving. In two experiments, adult participants were asked to verbally solve single-digit addition, subtraction and multiplication problems. Operands and arithmetic signs were presented sequentially. Although both O1 and arithmetic sign were presented at the center of the screen, O2 was displayed either in the left or right hemifield. We found that addition problems were solved faster when O2 was on the right than on the left side, whereas subtraction problems were solved faster when O2 was on the left than on the right side. These effects were observed up to 300 ms after the arithmetic sign. Within that time window, we did not find any operation-dependent spatial bias when O2 was equal to zero. No spatial bias was also observed when the operation was a multiplication problem. Therefore, our results demonstrate the presence of horizontal shifts of attention that are specific to single-digit (and non-zero) addition and subtraction problems.

4.1. Mental movements along the MNL

Overall, our findings are consistent with the idea that single-digit arithmetic problem-solving is associated with the activation of procedures that require moving to the right or left of a MNL. These procedures are likely to originate from the arithmetic sign itself. Indeed, both Fayol and Thevenot (2012) and Roussel et al. (2002) showed that previewing an addition or a subtraction sign speeds up the processing of a subsequent single-digit addition or subtraction problem, suggesting that these signs might recruit arithmetic procedures. We propose that arithmetic signs may activate a representation of the MNL, along which attention can be shifted rightward or leftward depending on the operator (addition or subtraction). Of course, movement along the MNL is entirely dependent upon O2. That is, no movement is necessary if O2 is equal to zero. We think that this explains why, 150 ms and 300 ms after the sign, no operation-dependent spatial bias could be observed when O2 is zero. Thus, the spatial biases captured here very likely arise from the combined processing of the arithmetic sign and O2.

4.2. Alternative explanations

Although our findings appear consistent with the idea of mental movements along the MNL, we review here several potential explanations for these effects. First, one might wonder whether the effects might be due to differences in the size of the operands across operations. This is impossible because we used the exact same operands for addition and subtraction problems in Experiment 1, as well as for addition and multiplication problems in Experiment 2. Thus, the differences in horizontal shifts of attention cannot stem from differences in the magnitude of operands across operations.

Second, it could be argued that differences in the size of the results contribute to the biases observed in Experiment 1 (i.e., results of subtraction problems were overall smaller than results of addition problems). However, this explanation is ruled out by Experiment 2. Indeed, we found a larger rightward bias for addition than multiplication problems, despite the fact that addition problems were associated with results of smaller sizes than multiplication problems. Furthermore, associations between numbers and space appear to occur later than the biases observed here (i.e., around 400 ms in Fischer et al., 2003) whereas our effects can be observed as early as 150 ms), also arguing against that explanation.

Third, it could be claimed that a simple association between arithmetic signs and space might account for our results. For instance, Pinhas et al. (2014) recently demonstrated that participants show a preference for the right (compared to the left) hand when classifying *plus* signs whereas they show a preference (albeit weaker) for the left (compared to the right) hand when evaluating *minus* signs, an effect termed *operation sign spatial association* (OSSA). Marghetis et al. (2014) further reported that arithmetic operators influenced horizontal hand trajectories when participants were required to choose the result of a problem with a mouse cursor. This explanation can also be ruled out. Indeed, if simple associations between arithmetic signs and space could explain the operation-dependent spatial bias observed for main arithmetic problems up to 300 ms after the arithmetic sign, it should also be observed with zero-problems within the same time window. However, we did not observe any such effect with these problems. Therefore, it is very unlikely that the biases observed for the main problems can be explained by simple associations between arithmetic signs and space. Note, however, that such an explanation may account for the effect observed for zero problems 450 ms after the sign (see Discussion of Experience 1).

Fourth, it has been speculated that arithmetic signs might be associated with generic metaphorical associations such as "more is right" and "less is left" (Hartmann et al., 2015; McCrink & Wynn, 2009). Such associations would arise with arithmetic practice and might provide a heuristic way to check whether a result obtained by rote or calculation "feels right" (i.e., results of addition problems should be larger than O1 and results of subtraction problems should be smaller than O1) (Hartmann et al., 2015; Marghetis et al., 2014). It might thus be argued that arithmetic signs might activate those heuristics, which in turn might lead to the effects we observe. This possibility is inconsistent with Experiment 2. Indeed, given that results of multiplication problems are also larger than (or at least as large as) O1, this view would predict that multiplication problems would also be associated with the right side of space. Our results did not show any rightward shift of attention associated with multiplication problems. Thus, our findings indicate that horizontal shifts of attention during addition and subtraction may reflect specific calculation procedures rather than generic intuitions about what type of result should be accepted or not.

4.3. Origin of movements along the MNL

Clearly, the idea that mental arithmetic is associated with spatial biases and that these biases reflect movements along the MNL has been suggested in several prior studies (Knops et al., 2009; Marghetis et al., 2014; Masson & Pesenti, 2014; Wiemers, Bekkering, & Lindemann, 2014). Yet, most of these studies investigated more complex arithmetic problems (e.g., double-digit) (Knops et al., 2009; Wiemers et al., 2014). Furthermore, the few studies that investigated spatial biases in single-digit problems have provided conflicting evidence for the presence of horizontal spatial biases *during* simple addition and subtraction (see Introduction) (Hartmann et al., 2015; Marghetis et al., 2014; Masson & Pesenti, 2014; Pinhas & Fischer, 2008). Our study contributes to this literature by showing that horizontal shifts of attention are elicited during problem-solving when participant solve single-digit addition and subtraction.

Why would arithmetic single-digit arithmetic problems be associated with mental movements along the MNL? One possibility is that these movements originate from the repeated practice of counting during arithmetic learning. For example, Baroody proposed that explicit procedures are progressively compacted and replaced by automatized ones through repetitive practice of arithmetic (Baroody, 1983, 1984, 1994). This hypothesis is in line with the idea that some cognitive procedures are compiled and implemented automatically and unconsciously (Anderson, 1982, 1983, 1987). Through practice, abstract procedures may become increasingly efficient and engaged automatically as soon as participants know the nature of the task to perform (Fayol & Thevenot, 2012; Roussel et al., 2002). In an arithmetic context, it is plausible that some algorithmic procedures (e.g., step-by-step internal counting) explicitly used by children when learning arithmetic are progressively internalized into rapid left–right attentional movement of the MNL.

4.4. Relevance for the debate about strategies in simple arithmetic

It is important to note that our study only demonstrates that procedures relying on movements along the MNL are *activated* during single-digit addition and subtraction problem-solving. It does not indicate whether these procedures are necessarily used by participants to solve these problems. In other words, our data do not directly speak to the question of whether single-digit addition and subtraction problems are solved by means of procedural or retrieval strategies. Nevertheless, activation of procedural strategies is of course a necessary condition for their subsequent use, and increasing evidence suggests that such procedures are indeed employed when solving even very simple problems. For example, Fayol and Thevenot (2012) and Roussel et al. (2002) demonstrated that the simple perception of an arithmetic sign before a single-digit addition and subtraction problem facilitates subsequent problem-solving, indicating that any procedural knowledge that is activated by these signs is used to solve these problems. More recently, Barrouillet and Thevenot (2013) showed that response times of addition problems involving operands from 1 to 4 increase linearly with the magnitude of the operands, suggesting that participants may use step-by-step counting procedures that would be fast and automatized (see also Groen and Parkman (1972) for data showing that adults solve single-digit addition problems with a slope of 20 ms per increment). Because such abstract procedures may be implemented automatically without ever reaching consciousness by adults, participants may be unable to report using them (Barrouillet & Thevenot, 2013; Fayol & Thevenot, 2012).

Our study further shows that the presence of procedural strategies during single-digit arithmetic is not restricted to experts, as suggested by some (Campbell, Chen, & Maslany, 2013; Chen & Campbell, 2015). Indeed, Fayol and Thevenot (2012) tested participants who were highly proficient in arithmetic: All of their participants scored between 70 and 145 (mean = 90; SD = 19.30) on the French kit, a test measuring arithmetic fluency for which the mean in a population is 59 (French, Ekstrom, & Price, 1963; Thevenot, Barrouillet, Castel, & Jimenez, 2011). In contrast, we recruited here volunteer students from the University of Lyon and did not select these participants based on arithmetic proficiency. 18 participants of our Experiment 2 agreed to complete the French Kit. Their mean score ranged from 28 to 144 (mean = 68; SD = 26), indicating a highly heterogeneous population. Therefore, our findings are clearly not restricted to participants with a particularly high level of arithmetic skills.

4.5. No spatial biases for multiplication

Finally, our study indicates that the activation of procedures involving movements along the MNL might be restricted to single-digit addition and subtraction. Indeed, we did not find any evidence for the presence of an attentional bias during multiplication problem-solving. Again, our results are consistent with that of Fayol and Thevenot (2012). The authors found that the presentation of addition and subtraction signs facilitated subsequent problem-solving, but no effect was found for multiplication signs. This is in keeping with the idea that single-digit multiplication problems may be predominantly solved by direct retrieval of arithmetic facts. Multiplication problems are unique among arithmetic problems in that they are explicitly learned by rote in school. They may thus be strongly related to an associative network of facts and might not be associated with procedural strategies in adults (Campbell & Xue, 2001; Dehaene & Cohen, 1995; Galfano et al., 2003; Ischebeck et al., 2006; Rusconi et al., 2004; Thibodeau et al., 1996). This is consistent with several recent neuroimaging studies showing that multiplication is associated with brain regions involved in verbal retrieval, whereas single-digit subtraction and addition is associated with brain regions involved in spatial attentional processing (Prado, Mutreja, & Booth, 2014; Prado et al., 2011; Zhou et al., 2007).

4.6. Conclusion

To summarize, the present study shows that solving single-digit addition and subtraction problems, but not multiplication problems, is accompanied by horizontal shifts of attention. This supports the idea that single-digit addition and subtraction, but not multiplication, is associated with the activation of procedures that may take the form of fast attentional movements along the MNL. Future studies are needed to investigate when these procedures emerge over development, the condition under which they are used to solve single-digit problems in adults, and to what extent this procedural knowledge relates to arithmetic skill.

Conflict of interest

The authors declare no competing financial interests.

Acknowledgments

This research was supported by a Grant from the European Union (Marie-Curie Carreer Integration Grant n° PCIG12-GA-2012-333602) to JP and a Grant from the French Ministry of Higher Education and Research to RM. We would like to thank Arthur J. Baroody and two anonymous reviewers for their helpful and constructive comments on a previous version of our manuscript.

Appendix A

Full list of arithmetic problems used in Experiment 1.

	Addition	Subtraction
Small problems	2 + 1	2 − 1
	3 + 1	3 − 1
	3 + 2	3 − 2
	4 + 1	4 − 1
	4 + 2	4 − 2
	4 + 3	4 − 3
	5 + 1	5 − 1
	5 + 2	5 − 2
	5 + 3	5 − 3
	5 + 4	5 − 4

(*continued on next page*)

Appendix A (*continued*)

	Addition	Subtraction
Large problems	6 + 5	6 − 5
	7 + 5	7 − 5
	7 + 6	7 − 6
	8 + 5	8 − 5
	8 + 6	8 − 6
	8 + 7	8 − 7
	9 + 5	9 − 5
	9 + 6	9 − 6
	9 + 7	9 − 7
	9 + 8	9 − 8
Zero problems	1 + 0	1 − 0
	2 + 0	2 − 0
	3 + 0	3 − 0
	4 + 0	4 − 0
	5 + 0	5 − 0
	6 + 0	6 − 0
	7 + 0	7 − 0
	8 + 0	8 − 0
	9 + 0	9 − 0

Appendix B

Full list of arithmetic problems used in Experiment 2.

	Addition		Multiplication	
Small problems	2 + 1	1 + 2	2 × 1	1 × 2
	3 + 1	1 + 3	3 × 1	1 × 3
	3 + 2	2 + 3	3 × 2	2 × 3
	4 + 1	1 + 4	4 × 1	1 × 4
	4 + 2	2 + 4	4 × 2	2 × 4
	4 + 3	3 + 4	4 × 3	3 × 4
	5 + 1	1 + 5	5 × 1	1 × 5
	5 + 2	2 + 5	5 × 2	2 × 5
	5 + 3	3 + 5	5 × 3	3 × 5
	5 + 4	4 + 5	5 × 4	4 × 5
Large problems	6 + 5	5 + 6	6 × 5	5 × 6
	7 + 5	5 + 7	7 × 5	5 × 7
	7 + 6	6 + 7	7 × 6	6 × 7
	8 + 5	5 + 8	8 × 5	5 × 8
	8 + 6	6 + 8	8 × 6	6 × 8
	8 + 7	7 + 8	8 × 7	7 × 8
	9 + 5	5 + 9	9 × 5	5 × 9
	9 + 6	6 + 9	9 × 6	6 × 9
	9 + 7	7 + 9	9 × 7	7 × 9
	9 + 8	8 + 9	9 × 8	8 × 9

References

Anderson, J. R. (1982). Acquisition of cognitive skill. *Psychological Review, 89*(4), 369–406. http://dx.doi.org/10.1037/0033-295X.89.4.369.

Anderson, J. R. (1983). *The architecture of cognition*. Cambridge, MA: Harvard University Press.

Anderson, J. R. (1987). Skill acquisition: Compilation of weak-method problem situations. *Psychological Review, 94*(2), 192–210. http://dx.doi.org/10.1037/0033-295X.94.2.192.

Ashcraft, M. H. (1982). The development of mental arithmetic: A chronometric approach. *Developmental Review, 2*(3), 213–236. http://dx.doi.org/10.1016/0273-2297(82)90012-0.

Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition, 44*(1), 75–106. http://dx.doi.org/10.1016/0010-0277(92)90051-I.

Ashcraft, M. H., & Guillaume, M. M. (2009). Mathematical cognition and the problem size effect. In B. Ross (Ed.). *The psychology of learning and motivation* (Vol. 51, pp. 121–151). Burlington: Academic Press. http://dx.doi.org/10.1016/S0079-7421(09)51004-3.

Baroody, A. J. (1983). The development of procedural knowledge: An alternative explanation for chronometric trends of mental arithmetic. *Developmental Review, 3*(2), 225–230. http://dx.doi.org/10.1016/0273-2297(83)90031-X.

Baroody, A. J. (1984). A reexamination of mental arithmetic models and data: A reply to Ashcraft. *Developmental Review, 4*(2), 148–156. http://dx.doi.org/10.1016/0273-2297(84)90004-2.

Baroody, A. J. (1994). An evaluation of evidence supporting fact-retrieval models. *Learning and Individual Differences, 6*(1), 1–36. http://dx.doi.org/10.1016/1041-6080(94)90013-2.

Barrouillet, P., Mignon, M., & Thevenot, C. (2008). Strategies in subtraction problem solving in children. *Journal of Experimental Child Psychology, 99*(4), 233–251. http://dx.doi.org/10.1016/j.jecp.2007.12.001.

Barrouillet, P., & Thevenot, C. (2013). On the problem-size effect in small additions: Can we really discard any counting-based account? *Cognition, 128*(1), 35–44. http://dx.doi.org/10.1016/j.cognition.2013.02.018.

Brysbaert, M. (1995). Arabic number reading: On the nature of the numerical scale and the origin of phonological recoding. *Journal of Experimental Psychology: General, 124*(4), 434–452. http://dx.doi.org/10.1037/0096-3445.124.4.434.

Campbell, J. I. D., Chen, Y., & Maslany, A. J. (2013). Retrieval-induced forgetting of arithmetic facts across cultures. *Journal of Cognitive Psychology, 25*(6), 759–773. http://dx.doi.org/10.1080/20445911.2013.820191.

Campbell, J. I. D., & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology: General, 130*(2), 299–315. http://dx.doi.org/10.1037/0096-3445.130.2.299.

Chen, Y., & Campbell, J. I. (2015). Operator and operand preview effects in simple addition and multiplication: A comparison of Canadian and Chinese adults. *Journal of Cognitive Psychology*, 1–9. http://dx.doi.org/10.1080/20445911.2014.999685 (ahead-of-print).

Chen, Q., & Verguts, T. (2012). Spatial intuition in elementary arithmetic: A neurocomputational account. *PLoS ONE, 7*(2), e31180. http://dx.doi.org/10.1371/journal.pone.0031180.

Cooney, J. B., Swanson, H. L., & Ladd, S. F. (1988). Acquisition of mental multiplication skill: Evidence for the transition between counting and retrieval strategies. *Cognition and Instruction, 5*(4), 323–345. http://dx.doi.org/10.1207/s1532690xci0504_5.

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General, 122*(3), 371–396. http://dx.doi.org/10.1037//0096-3445.122.3.371.

Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition, 1*(1), 83–120.

Fayol, M., & Thevenot, C. (2012). The use of procedural knowledge in simple addition and subtraction problems. *Cognition, 123*(3), 392–403. http://dx.doi.org/10.1016/j.cognition.2012.02.008.

Fischer, M. H., Castel, A. D., Dodd, M. D., & Pratt, J. (2003). Perceiving numbers causes spatial shifts of attention. *Nature Neuroscience, 6*(6), 555–556. http://dx.doi.org/10.1038/nn1066.

Fischer, M. H., & Shaki, S. (2014a). Spatial associations in numerical cognition – From single digits to arithmetic. *The Quarterly Journal of Experimental Psychology, 67*(8), 1461–1483. http://dx.doi.org/10.1080/17470218.2014.927515.

Fischer, M. H., & Shaki, S. (2014b). Spatial biases in mental arithmetic. *The Quarterly Journal of Experimental Psychology, 67*(8), 1457–1460. http://dx.doi.org/10.1080/17470218.2014.927516.

Forster, K. I., & Forster, J. C. (2003). DMDX: A windows display program with millisecond accuracy. *Behavior Research Methods, Instruments, & Computers, 35*(1), 116–124. http://dx.doi.org/10.3758/BF03195503.

French, J. W., Ekstrom, R. B., & Price, I. A. (1963). *Kit of reference tests for cognitive factors*. Princeton, NJ: Educational Testing S.

Galfano, G., Rusconi, E., & Umiltà, C. (2003). Automatic activation of multiplication facts: Evidence from the nodes adjacent to the product. *The Quarterly Journal of Experimental Psychology: Section A, 56*(1), 31–61. http://dx.doi.org/10.1080/02724980244000332.

Geary, D. C. (1994). *Children's mathematical development: Research and practical applications*. Washington, DC: American Psychological Association.

Geary, D. C., & Burlingham-Dubree, M. (1989). External validation of the strategy choice model for addition. *Journal of Experimental Child Psychology, 47*(2), 175–192. http://dx.doi.org/10.1016/0022-0965(89)90028-3.

Geary, D. C., Frensch, P. A., & Wiley, J. G. (1993). Simple and complex mental subtraction: Strategy choice and speed-of-processing differences in younger and older adults. *Psychology and Aging, 8*(2), 242–256. http://dx.doi.org/10.1037//0882-7974.8.2.242.

Groen, G. J., & Parkman, J. M. (1972). A chronometric analysis of simple addition. *Psychological Review, 79*(4), 329–343. http://dx.doi.org/10.1037/h0032950.

Hartmann, M., Mast, F. W., & Fischer, M. H. (2015). Spatial biases during mental arithmetic: Evidence from eye movements on a blank screen. *Frontiers in Psychology, 6*, 1–8. http://dx.doi.org/10.3389/fpsyg.2015.00012.

Howell, D. C. (2011). *Statistical methods for psychology*. Belmont, CA: Wadsworth Cengage Learning.

Hubbard, E. M., Piazza, M., Pinel, P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Reviews Neuroscience, 6*(6), 435–448. http://dx.doi.org/10.1038/nrn1684.

Ischebeck, A., Zamarian, L., Siedentopf, C., Koppelstätter, F., Benke, T., Felber, S., & Delazer, M. (2006). How specifically do we learn? Imaging the learning of multiplication and subtraction. *Neuroimage, 30*(4), 1365–1375. http://dx.doi.org/10.1016/j.neuroimage.2005.11.016.

Klein, E., Huber, S., Nuerk, H.-C., & Möller, K. (2014). Operational momentum affects eye fixation behaviour. *The Quarterly Journal of Experimental Psychology, 67*(8), 1614–1625. http://dx.doi.org/10.1080/17470218.2014.902976.

Knops, A., Dehaene, S., Berteletti, I., & Zorzi, M. (2014). Can approximate mental calculation account for operational momentum in addition and subtraction? *The Quarterly Journal of Experimental Psychology, 67*(8), 1541–1556. http://dx.doi.org/10.1080/17470218.2014.890234.

Knops, A., Thirion, B., Hubbard, E. M., Michel, V., & Dehaene, S. (2009). Recruitment of an area involved in eye movements during mental arithmetic. *Science, 324*(5934), 1583–1585. http://dx.doi.org/10.1126/science.1171599.

Knops, A., Viarouge, A., & Dehaene, S. (2009). Dynamic representations underlying symbolic and nonsymbolic calculation: Evidence from the operational momentum effect. *Attention, Perception, & Psychophysics, 71*(4), 803–821. http://dx.doi.org/10.3758/APP.71.4.803.

Knops, A., Zitzmann, S., & McCrink, K. (2013). Examining the presence and determinants of operational momentum in childhood. *Frontiers in Psychology, 4*, 1–14. http://dx.doi.org/10.3389/fpsyg.2013.00325.

LeFevre, J.-A., Sadesky, G. S., & Bisanz, J. (1996). Selection of procedures in mental addition: Reassessing the problem size effect in adults. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 22*(1), 216–230. http://dx.doi.org/10.1037/0278-7393.22.1.216.

Logan, G. D. (1988). Toward an instance theory of automatization. *Psychological Review, 95*(4), 492–527. http://dx.doi.org/10.1037/0033-295X.95.4.492.

Marghetis, T., Núñez, R., & Bergen, B. K. (2014). Doing arithmetic by hand: Hand movements during exact arithmetic reveal systematic, dynamic spatial processing. *The Quarterly Journal of Experimental Psychology, 67*(8), 1579–1596. http://dx.doi.org/10.1080/17470218.2014.897359.

Masson, N., & Pesenti, M. (2014). Attentional bias induced by solving simple and complex addition and subtraction problems. *The Quarterly Journal of Experimental Psychology, 67*(8), 1514–1526. http://dx.doi.org/10.1080/17470218.2014.903985.

McCrink, K., Dehaene, S., & Dehaene-Lambertz, G. (2007). Moving along the number line: Operational momentum in nonsymbolic arithmetic. *Perception & Psychophysics, 69*(8), 1324–1333. http://dx.doi.org/10.3758/BF03192949.

McCrink, K., & Wynn, K. (2009). Operational momentum in large-number addition and subtraction by 9-month-olds. *Journal of Experimental Child Psychology, 103*(4), 400–408. http://dx.doi.org/10.1016/j.jecp.2009.01.013.

Núñez-Peña, M. I., Gracia-Bafalluy, M., & Tubau, E. (2011). Individual differences in arithmetic skill reflected in event-related brain potentials. *International Journal of Psychophysiology, 80*(2), 143–149. http://dx.doi.org/10.1016/j.ijpsycho.2011.02.017.

Pinhas, M., & Fischer, M. H. (2008). Mental movements without magnitude? A study of spatial biases in symbolic arithmetic. *Cognition, 109*(3), 408–415. http://dx.doi.org/10.1016/j.cognition.2008.09.003.

Pinhas, M., Shaki, S., & Fischer, M. H. (2014). Heed the signs: Operation signs have spatial associations. *The Quarterly Journal of Experimental Psychology, 67*(8), 1527–1540. http://dx.doi.org/10.1080/17470218.2014.892516.

Pinhas, M., Shaki, S., & Fischer, M. H. (2015). Addition goes where the big numbers are: Evidence for a reversed operational momentum effect. *Psychonomic Bulletin & Review, 22*(4), 993–1000. http://dx.doi.org/10.3758/s13423-014-0786-z.

Prado, J., Mutreja, R., & Booth, J. R. (2014). Developmental dissociation in the neural responses to simple multiplication and subtraction problems. *Developmental Science, 17*(4), 537–552. http://dx.doi.org/10.1111/desc.12140.

Prado, J., Mutreja, R., Zhang, H., Mehta, R., Desroches, A. S., Minas, J. E., & Booth, J. R. (2011). Distinct representations of subtraction and multiplication in the neural systems for numerosity and language. *Human Brain Mapping, 32*(11), 1932–1947. http://dx.doi.org/10.1002/hbm.21159.

Protopapas, A. (2007). Check Vocal: A program to facilitate checking the accuracy and response time of vocal responses from DMDX. *Behavior Research Methods, 39*(4), 859–862. http://dx.doi.org/10.3758/BF03192979.

Ric, F., & Muller, D. (2012). Unconscious addition: When we unconsciously initiate and follow arithmetic rules. *Journal of Experimental Psychology: General, 141*(2), 222–226. http://dx.doi.org/10.1037/a0024608.

Robinson, K. M. (2001). The validity of verbal reports in children's subtraction. *Journal of Educational Psychology, 93*(1), 211–222. http://dx.doi.org/10.1037/0022-0663.93.1.211.

Roussel, J.-L., Fayol, M., & Barrouillet, P. (2002). Procedural vs. direct retrieval strategies in arithmetic: A comparison between additive and multiplicative problem solving. *European Journal of Cognitive Psychology, 14*(1), 61–104. http://dx.doi.org/10.1080/09541440042000115.

Rusconi, E., Galfano, G., Speriani, V., & Umiltà, C. (2004). Capacity and contextual constraints on product activation: Evidence from task-irrelevant fact retrieval. *The Quarterly Journal of Experimental Psychology: Section A, 57*(8), 1485–1511. http://dx.doi.org/10.1080/02724980343000873.

Seyler, D. J., Kirk, E. P., & Ashcraft, M. H. (2003). Elementary subtraction. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 29*(6), 1339–1352. http://dx.doi.org/10.1037/0278-7393.29.6.1339.

Siegler, R. S. (1996). *Emerging minds: The process of change in children's thinking*. New-York: Oxford University Press.

Siegler, R. S., & Shipley, C. (1995). Variation, selection, and cognitive change. In G. Halford & T. Simon (Eds.), *Developing cognitive competence: New approaches to process modeling* (pp. 31–76). Hillsdale, NJ: Erlbaum.

Siegler, R. S., & Shrager, J. (1984). Strategy choices in addition and subtraction: How do children know what to do? In C. Sophian (Ed.), *Origins of cognitive skills* (pp. 229–293). Hillsdale, NJ: Erlbaum.

Thevenot, C., Barrouillet, P., Castel, C., & Jimenez, S. (2011). Better elementary number processing in higher skill arithmetic problem solvers: Evidence from the encoding step. *The Quarterly Journal of Experimental Psychology, 64*(11), 2110–2124. http://dx.doi.org/10.1080/17470218.2011.582129.

Thevenot, C., Barrouillet, P., & Fayol, M. (2001). Algorithmic solution of arithmetic problems and operands-answer associations in long-term memory. *The Quarterly Journal of Experimental Psychology: Section A, 54*(2), 599–611. http://dx.doi.org/10.1080/713755966.

Thevenot, C., Fanget, M., & Fayol, M. (2007). Retrieval or nonretrieval strategies in mental arithmetic? An operand recognition paradigm. *Memory & Cognition, 35*(6), 1344–1352. http://dx.doi.org/10.3758/BF03193606.

Thibodeau, M. H., Lefevre, J.-A., & Bisanz, J. (1996). The extension of the interference effect to multiplication. *Canadian Journal of Experimental Psychology, 50*(4), 393–396. http://dx.doi.org/10.1037/1196-1961.50.4.393.

Tzelgov, J., Ganor-Stern, D., & Maymon-Schreiber, K. (2009). The representation of negative numbers: Exploring the effects of mode of processing and notation. *The Quarterly Journal of Experimental Psychology, 62*(3), 605–624. http://dx.doi.org/10.1080/17470210802034751.

Wiemers, M., Bekkering, H., & Lindemann, O. (2014). Spatial interferences in mental arithmetic: Evidence from the motion-arithmetic compatibility effect. *The Quarterly Journal of Experimental Psychology, 67*(8), 1557–1570. http://dx.doi.org/10.1080/17470218.2014.889180.

Wood, G., Willmes, K., Nuerk, H.-C., & Fischer, M. H. (2008). On the cognitive link between space and number: A meta-analysis of the SNARC effect. *Psychology Science Quarterly, 50*(4), 489–525.

Zhou, X., Chen, C., Zang, Y., Dong, Q., Chen, C., Qiao, S., & Gong, Q. (2007). Dissociated brain organization for single-digit addition and multiplication. *Neuroimage, 35*(2), 871–880. http://dx.doi.org/10.1016/j.neuroimage.2006.12.017.